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Sensitivity of DEA models to measurement errors

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Abstract

One of the weak points of DEA (*Data Envelopment Analysis*) models indicated in literature [1,2] is their sensitivity to variable measurement errors. The occurrence of data interference, which is the basis of the productivity analysis, may distort the classification of the units and may cause misjudgement of their effectiveness.

In the article the results of simulation concerning the DEA models sensitivity to occurrence and features of random element in the monitored variables describing the model are presented.

The set of thirty DMU (*Decision Making Units*) described by the means of three input variables, two output variables and one environmental variable was analysed. On the basis of the determined initial value of all the kinds of variables for each DMU, their relative effectiveness and their ranking were determined. Then, the value of each variable was interfered randomly with the noise with normal distribution N(m,σ) and once again relative effectiveness and ranking of DMU were determined. The calculation was done repeatedly, taking into account different levels of variance. The simulation carried out in the described manner was the basis for the assessment of the stability of the classification with the occurrence of measurement errors.

On the basis of the research, the limits of DEA models resistance to the occurrence of errors in the data that are used for productivity analysis were determined.

In the authors' opinion, the proposals in the article may be recognised as a vital input for the development of the methodology of comparative productivity analysis by the means of DEA models.

1. Introduction

Market economy forces businessmen to undertake actions aiming at increasing productivity, that is: effectiveness of their activity, ability of reaching aims, realisation of strategy thanks to marketing, production, finance and management effectiveness measured by the ratio of inputs to outputs. In order to take effective actions, decision makers have to evaluate constantly their results and compare them to those of the competitors. The direct comparison of the

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quantitative data does not provide full information. The detailed analysis can be made by means of benchmarking tools.

Benchmarking, as a method of development and improvement of organisation actions that bases on confronting the own level of productivity with the results of other enterprises and organisations, is an effective tool in the hands of decision makers [1]. In the group of benchmarking tools the method Data Development Analysis is included. The method was established by Charnes, Cooper and Rhodes in 1978 in order to determine effective or productive units, serving as a model for the others. The analysed units are Decision Making Units (DMU) that are able to generate effects: production, trade and service enterprises and public sector organisations. These units exist in the same market environment. They have common analysis factors that characterise their activity, with the exception for the differences in the scope and intensity of their applying. There are many factors describing processes that take place in the enterprise. Taking into consideration the number of variables describing DMU activity, in the DEA method multidimensional system may be employed, considering n DMU, which produce x effects (outputs) with the usage of y inputs [2].

$$P = \frac{\sum_{j=1}^{J} v_{m,j} x_{m,j}}{\sum_{i}^{I} u_{m,i} y_{m,i}},$$

J – number of inputs,
$v_{m,j}$ – weight of the j-input,
$y_{m,j}$ – output,
I – number of outputs,
$u_{m,i}$ – i-weight of i-output,
$x_{m,i}$ – i-output.
m – index of DMU

The effect of the method is the ranking of DMU according to the effectiveness coefficient. Data Envelopment Analysis bases on the principles of linear programming with the optimization of weights, separately for each unit. Attribution of weights to each variable reveals strong points of the unit. Since DMU exist in different conditions they have to adapt to the environment. More difficult market conditions do not mean that the units with for example lower profits are less productive than the others. Effectiveness of resource management is vital.

Very often the conditions are described by means of qualitative variables, not included in quantative methods. Data Envelopment Analysis enables considering environmental variables and uncontrolled variables, which at the same time leads to reduction of random factors. It makes the method flexible and possible to implement in the analysis of units of different profile of activity. This is the reason for existence of many models in the method. There are classical models like CCR and BCC, additive models, multiplicative models and further modifications and extensions aiming at better illustration of the processes occurring in DMU [3].

2. Methodology of research

One of the weak sides of DEA models is sensitivity to variable measurement errors [4,5]. Interest in the topic can be observed in world literature. It was studied how number of DMU, number of inputs and outputs, choice of the model and change of data or variables influence sensitivity and stability of the method [6,7]. The attempt of study, how data interference influences DEA calculation results, was made. It is a vital issue because of occurrence of random element in different kinds of measurement.

The set of thirty DMU described by means of three input variables, two output variables and one environmental variable was analysed (Fig. 1).

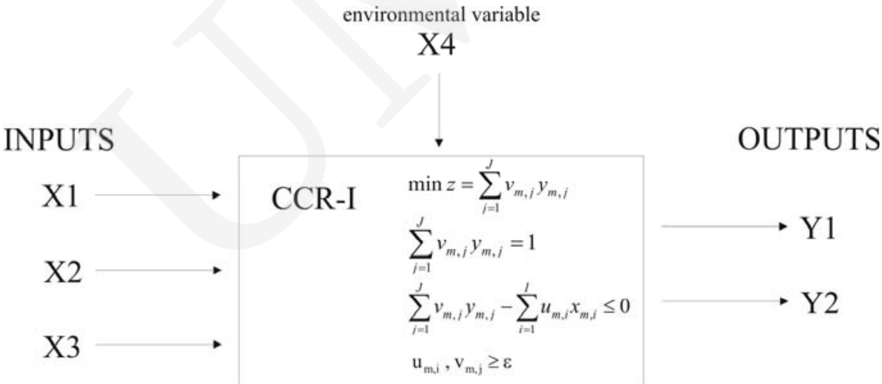


Fig. 1. Parameters of the analysed model. Source: Own study

Later basic statistics of variables, mean and standard variations were determined (Table 1).

Table 1. Results of statistics of analysed variables. Source: Own study

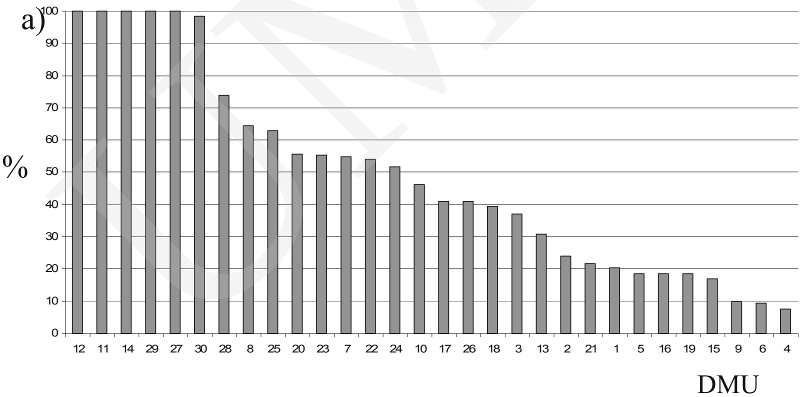
Inputs/Outputs	Mean	Standard variation	Variation coefficient
X1	23758.47	6342.26	0.266947
X2	128.5	31.99	0.248949
X3	17796.03	8217.11	0.461738
X4	35219.13	6597.72	0.187333
Y1	5010.33	3491.97	0.696954
Y2	20613.7	29025.94	0.710182

On the basis of initial values of all kinds of variables for each DMU, relative effectiveness and their basic ranking were established.

Later each variable was randomly interfered with the noise of interference with the parameters $N(\overline{x}_{m,k}, \overline{x}_{m,k} v)$ where m – index of DMU, k – number of variable. Different values of variable coefficient were considered, the same for all interfered variables.

In the first step, all variables were interfered with the variation coefficient (v) at the level of 0.01. Next, ranking of interfered variables was determined (Fig. 2). The procedure was repeated with the usage of variation coefficient at the level 0.05, 0.1, 0.15, 0.20, 0.25, 0.30.

The operation was done repeatedly for the interfered input variables: X1, X2, X3, the output variables X5 and X6, and then for the environmental variable X4 and one input variable X1. For all the interference relative effectiveness and DMU ranking was established.



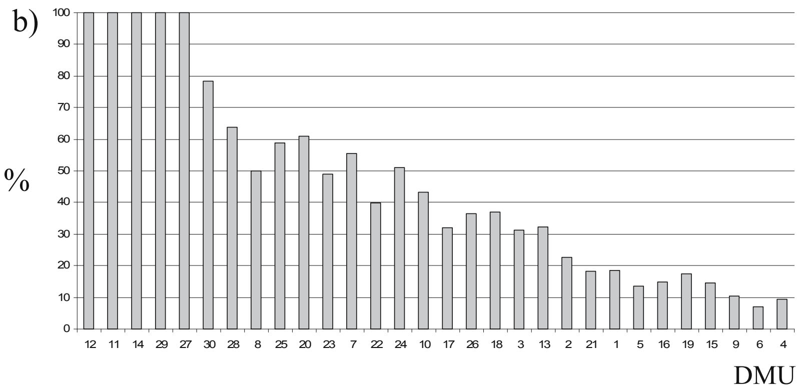
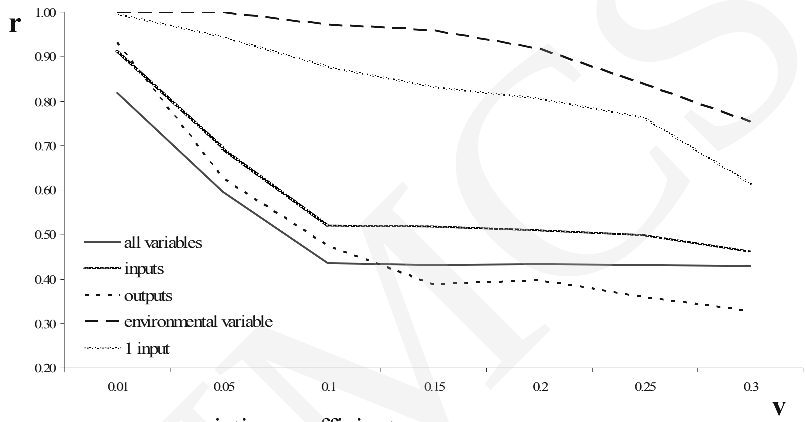


Fig. 2. a) Basic ranking, b) Example of DEA ranking result with the variation coefficient at the level 0.15 for the interfered input variables. Source: Own study

To determine the level of ranking interference, nonparametric measure of correlation between the basic ranking and that established as a result of interference Spearman's rank correlation coefficient (for each group of interference) was used (Table 2).



v – variation coefficient
r – Spearman's rank correlation coefficient

Fig. 3. Graphical presentation of results. Source: Own study

In Figure 3, the linear relation between the variation coefficient and the Spearman's rank correlation coefficient is presented. Interference of two or more variables causes changes in results even with the data variation coefficient of 1%.

Conclusions

This research is a part of in-depth investigation of DEA method application. An attempt of studying DEA models sensitivity to the measurement errors was made. To assess stability of the ranking Spearman's rank correlation coefficient was used.

As a result of simulation it was assumed that for the analysed data the method is sensitive to measurement errors. The simulation illustrates the level of sensitivity of DEA. The models show the lowest stability when all variables are interfered. Interference of input and output variables highly effects the results. It turns out that the models are more resistant to interference of environmental variable. It can be an effect of a different way of taking into account this kind of variables as far as calculation is concerned.

The level of Spearman's rank correlation coefficient has different values depending on the data. The less diversified data, the higher sensitivity to changes. The approach of the research provides the way of verifying the rankings. At the same time high coefficients are the basis of the statement that the results are reliable. On the other hand, awareness of Spearman's coefficient fast decrease persuades to pay special attention to the choice of reliable data.

The simulations also established more stability of the part of effective units and those considered as more effective than the others in the ranking. It means that units considered effective in the basic ranking, as a result of interference were still holding their positions as effective units. Analogous situation can be observed as far as units that hold final positions in the ranking are concerned. It indicates directions for further research.

In the authors' opinion the proposals presented in the article may serve as a vital input to the methodology of comparative productivity analysis by means of DEA models.

References

[1] Nazarko J., Chrabołowska J., *Benchmarking in the effectiveness assessment of national power distribution systems*. Taksonomy, 12 (2005).

[2] Ramanathan R., *An Introduction to Data Envelopment Analysis. A Tool for Performance Measurement*, Sage Publications, New Delhi, Thousand Oaks, London, (2003).

[3] Coelli T., Rao D.S.P., Battese G.E., *An introduction to efficiency and productivity analysis*. Kluwer Academic Publishers, Boston, (2002).

[4] Seiford L.M., Zhu J., *Sensitivity and Stability of the Classifications of Returns to Scale in Data Envelopment Analysis*. Journal of Productivity Analysis, 12(1) (1999).

[5] Cooper W.W., Seiford L.M., Zhu J., *Handbook on Data Envelopment Analysis*. Kluwer Academic Publisher**,** Boston, **(**2004).

[6] Avkiran N.K., *Stability and integrity tests in data envelopment analysis*. Socio-Economic Planning Sciences, available online 10 August 2006.

[7] Zhu J., *Quantitative models for performance evaluation and benchmarking*. Springer, USA, (2003).