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SPACE-TIME CURVATURE OF GENERAL RELATIVITY AND ENERGY DENSITY OF A THREE-DIMENSIONAL QUANTUM VACUUM

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ABSTRACT

A three-dimensional quantum vacuum condensate is introduced as a fundamental medium from which gravity emerges in a geometro-hydrodynamic limit. In this approach, the curvature of space-time characteristic of general relativity is obtained as a mathematical value of a more fundamental actual variable energy density of quantum vacuum which has a concrete physical meaning. The fluctuations of the quantum vacuum energy density suggest an interesting solution for the dark energy problem.

Keywords: curvature of space, general relativity, energy density of quantum vacuum, dark energy.

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1. INTRODUCTION

The 20th century theoretical physics brought the idea of a unified quantum vacuum as a fundamental medium subtending the observable forms of matter, energy and space-time. The notion of an "empty" space devoid of any physical properties has been replaced with that of a quantum vacuum state, defined to be the ground (lowest energy density) state of a collection of quantum fields. A peculiar and truly quantum mechanical feature of the quantum fields of the vacuum is that they exhibit zero-point fluctuations everywhere in space, even in regions which are devoid of matter and radiation. These zero-point fluctuations of the quantum fields, as well as other "vacuum phenomena" of quantum field theory, give rise to an enormous vacuum energy density.

The existence of a physical vacuum can be considered as the most important consequence of contemporary quantum field theories, such as the quantum electrodynamics, the Weinberg-Salam-Glashow theory of electroweak interactions and the quantum chromodynamics of strong interactions. These quantum field theories imply that various contributions to the vacuum energy density exist: the fluctuations characterizing the zero-point field, the fluctuations characterizing the quantum chromodynamic level of subnuclear physics, the fluctuations linked with the Higgs field, as well as perhaps other contributions from possible existing sources outside the Standard Model (for instance, Grand Unified Theories, string theories, etc.). On the other hand, there is no structure within the Standard Model which suggests no relations between the different contributions to the quantum vacuum energy density, and it is therefore customary to assume that the total vacuum energy density is, at least, as large as any of these individual contributions. As regards the role of the different contributions to the vacuum energy density, the reader can find a detailed analysis, for example, in the paper [1] by Rugh and Zinkernagel, who studied the connection between the vacuum concept in quantum field theory and the conceptual origin of the cosmological constant problem, and in the paper [2] by Timashev, who examined the possibility of considering the physical vacuum as a unified system governing the processes taking place in microphysics and macrophysics, which manifests itself on all space-time scales, from subnuclear to cosmological.

The realistic concept of the vacuum can be considered as the ultimate visiting card which completes and complements Einstein's theory of relativity. Relativity theory views space-time as a relative and dynamic manifold, interacting with matter and energy. It is the "background" against which the events of the manifest world unfold. But the origins of this background are not accounted for in relativity theory: space-time is simply "given" together with matter and energy. In general relativity, the standard interpretation of phenomena in gravitational fields is in terms of a fundamentally curved space-time. However, this approach leads to well-known problems if one aims to find an unifying picture which takes into account some basic aspects of the quantum theory. In order to escape this situation of impasse, several authors advocated thus alternative ways in order to treat gravitational interaction, in which the space-time manifold can be considered as an emergence of the deepest processes situated at the fundamental level of quantum gravity. In this regard, the germinal proposal of Sacharov was of deducing gravitation as a "metric elasticity" of space, which consists in a generalized force opposing the curving of space [3] (the reader can also see the reference [4] for a review of this concept). Sacharov's model starts from the interpretation of the action of space-time as the effect of quantum fluctuations of the vacuum in a curved space. Other interesting approaches are Haisch's and Rueda's model [5], regarding the interpretation of inertial mass and gravitational mass as effects of an electromagnetic quantum vacuum, Puthoff's polarizable vacuum model of gravitation [6] and, more recently, a model developed by Consoli based on ultra-weak excitations in a condensed manifold in order to describe gravitation and Higgs mechanism [7-9]. Under the construction of all of these models there is probably one underlying fundamental observation: as light in Euclid space deviates from a straight line in a medium with variable density, an "effective" curvature might originate, under opportune conditions, from the same physical flat-space vacuum.

In this paper, by following the philosophy that is at the basis of these approaches, we suggest a model of a three-dimensional (3D) quantum vacuum in which general relativity emerges as the hydrodynamic limit of some underlying theory of a more fundamental microscopic structure of space-time. According to this model, the curvature of space-time characteristic of general relativity can be considered as a mathematical value of a more fundamental actual energy density of quantum vacuum which has a concrete physical meaning. In the outer intergalactic space, namely in the absence of material objects, the energy density of the 3D quantum vacuum is defined by the following relation:

$$\rho_{pE} = \frac{m_p \cdot c^2}{l_p^3}, \tag{1}$$

where m_P is Planck's mass, c is the light speed and l_p is Planck's length. The quantity (1) is the maximum value of the quantum vacuum energy density and physically corresponds to the total average volumetric energy density, owed to all the frequency modes possible within the visible size of the universe, expressed by

$$\rho_{pE} = \sqrt{\frac{c^{14}}{\hbar^2 G^4}} \approx 4{,}641266 \cdot 10^{113}\, J/m^3 \cong 10^{97}\, Kg/m^3, \tag{2}$$

$\hbar$ being Planck's reduced constant, G the universal gravitation constant. In the outer intergalactic space curvature of space is zero and its energy density corresponds to the value (2). On the other hand, in the picture of Rueda's and Haisch's interpretation of the inertial mass as an effect of the electromagnetic quantum vacuum [5], the presence of a particle with a volume V_0 expels from the vacuum energy within this volume exactly the same amount of energy as is the particle's internal energy (equivalent to its rest mass). On the basis of Rueda's and Haisch's results, here we assume that each elementary particle is associated with fluctuations of the quantum vacuum which determine a diminishing of the quantum vacuum energy density. Therefore, one can say that in the presence of a material object the curvature of space increases and corresponds physically to a more fundamental diminishing of the energy density of the quantum vacuum, which, in the centre of the material object, is given by relation

$$\rho_{qvE} = \rho_{pE} - \frac{m \cdot c^2}{V}, \tag{3}$$

m and V being the mass and volume of the object [10]. Here, we propose that the quantum vacuum energy density is the fundamental, ultimate physical reality characterizing the gravitational space. The physical property of mass is considered as a secondary ontologically reality with respect to the energy density of quantum vacuum: the density of a given material object is produced by a change of the quantum vacuum energy density on the basis of equation

$$\rho_{mat} = \frac{\Delta\rho_{qvE}}{c^2}, \tag{4}$$

where $\Delta\rho_{qvE} = \rho_{pE} - \rho_{qvE}$.

This paper is structured in the following manner. In chapter 2 we will explore in what sense general relativity can be seen as the hydrodynamic limit of an underlying quantum vacuum condensate having quantized features. In chapter 3 we will show how the changes of the energy density of the 3D quantum vacuum give rise to the curvature of space-time characteristic of general relativity and which is similar to the curvature produced by a "dark energy" density. Finally, in chapter 4 we will analyse the motion of a material object in the curved space determined by the changes and fluctuations of the quantum vacuum energy density.

2. SPACE AS THE GEOMETRO-HYDRODYNAMIC LIMIT OF A 3D QUANTUM VACUUM CONDENSATE HAVING A DISCRETE NATURE

Taking account of Sacharov's assumption that the action of spacetime

$$S(R) = -\frac{1}{16\pi G}\int dx\sqrt{-gR}, \tag{5}$$

where R is the invariant Ricci tensor, is viewed as a change in the action of quantum fluctuations of vacuum in a curved space and considering the consistent histories approach of quantum mechanics [11-13], according to which the quantum evolution can be seen as the coherent superposition of virtual fine–grained histories, general relativity can be interpreted as the hydrodynamic limit of an underlying theory of "microscopic" structure of space, more precisely of a 3D quantum vacuum condensate whose most universal physical property is its energy density.

A fine-grained history can be defined by the value of a field $\Phi(x)$ at the point x and has quantum amplitude $\Psi[\Phi] = e^{iS[\Phi]}$, where S is the classical action corresponding to the considered history. The quantum interference between two virtual histories A and B can be quantified by a "decoherence" functional:

$$D_F[\Phi_A, \Phi_B] \approx \Psi[\Phi_A]\Psi[\Phi_B]^* \approx e^{i(S[\Phi_A]-S[\Phi_B])} \tag{6}$$

that gives the coarse-grained histories corresponding to the observations in classical world. The quantum amplitude for a coarse-grained history is then defined by:

$$\Psi[\omega] = \int D_F\Phi e^{iS}\omega[\Phi], \tag{7}$$

where ω can be considered as a "filter" function that selects which fine-grained histories are associated to the same superposition with their relative phases. The decoherence functional for a couple of coarse-grained histories is then:

$$D_F[\omega_A, \omega_B] = \int D_F\Phi_A D_F\Phi_B e^{i(S[\Phi_A]-S[\Phi_B])}\omega[\Phi_A]\omega[\Phi_B]^* \tag{8}$$

in which the histories Φ_A and Φ_B assume the same value at a given time instant of the future, where decoherence indicates that the different histories contributing to the full quantum evolution can exist individually, are characterized by quantum amplitude and that the system undergoes an information and predictability degradation [13] (in this sense the system becomes stochastic and dissipative). By applying the formalism (8) to hydrodynamics variables [14], Einstein's stress-energy tensor can be expressed through the following operator:

$$\hat{T}_{\mu\nu}\left(x_A, x_B\right) = \Gamma_{\mu\nu}\Phi\left(x_A\right)\Phi\left(x_B\right). \tag{9}$$

In equation (9) $\Gamma_{\mu\nu}$ is a generic field operator defined at two points that leads to the "conservation law":

$$\hat{T}_{\mu\nu}^{;\nu} = 0 \tag{10}$$

meaning that the decohered quantities, showing a classical behavior, are the conserved ones. It can be shown that, for an action $S\left[\Phi^l\right] = \Phi^l \Delta_{lm} \Phi^m$, the following relation holds

$$D_F\left[\hat{T}_{\mu\nu}^A, \hat{T}_{\mu\nu}^B\right] = \int D_F K_n^{\mu\nu}\left(x_A, x_B\right) \int D_F \Phi^l e^{i\Phi^l\left[\Delta + K_n^{\mu\nu}(x_A, x_B)\Gamma_{\mu\nu}^n(x_A, x_B)\right]_{lm}\Phi^m} e^{iK_n^{\mu\nu}(x_A, x_B) T_{\mu\nu}^n(x_A, x_B)} \approx e^{i\Omega\left[\hat{T}_{\mu\nu}^A(x_A, x_B), \hat{T}_{\mu\nu}^B(x_A, x_B)\right]} \tag{11}$$

in which we have used the integral representation of delta and the CTP indices $l.m.n = 1,2$, Ω being the closed–time path two-particle irreducible action.

The conservation of $\hat{T}_{\mu\nu}$ implies that the decoherence functional has maximum values in correspondence of the hydrodynamic variables (ρ, p) that, in turn, are the most readily decohered and have the highest probability to become classical. By applying the above procedure to Einstein's tensor $G_{\mu\nu}$ an analogy emerges between the conservation law for $\hat{T}_{\mu\nu}$ and the Bianchi identity $G_{\mu\nu}^{;\nu} = 0$ which implies the decoherence and the emergence of the hydrodynamics variables of the geometry. In this sense general relativity can be considered as geometro–hydrodynamics and the most readily decohered variables are those associated to the largest "inertia" representing the collective variables of geometry.

If general relativity must be regarded as a geometro-hydrodynamic limit of an underlying "microscopic" background where one has collective variables, and the laws governing macro-classical space-time are expressed in terms of collective variables, the precise characterization of this underlying background and thus the quantization of the general-relativistic metric or the connection variables will only result in the discovery of the excitations in the geometry and not of its quantum micro-structures. If we consider the collective hydrodynamics variables ρ and p appearing in the stress-energy tensor $T_{\mu\nu}$, then the quantization has sense when performed on the field function $\Phi(x)$ from which they are constructed and not on ρ and p themselves. The situation is similar to that regarding condensed matter physics in which the quantization of collective excitations leads to phonons and not to the atomic structure of matter. In the view of general relativity as geometro-hydrodynamic limit of an underlying background, there is therefore an important analogy between quantum to classical transition of gravity and the behavior of condensed matter. Moreover, given the collective variables (the metric and the connections in general relativity), how can we characterize the microscopic structure of the underlying background, namely what can we say about the quantum micro-structure from which the collective variables derive? In this regard, a possible strategy is of starting from a suitable theory of quantum microscopic structure and studying its previsions in the long wavelength–low energy limit. An approach of this kind has been recently suggested, for example, by Consoli [7-9], who has introduced a physical vacuum intended as a superfluid medium – a Bose condensate of elementary spinless quanta – whose long-range fluctuations, on a coarse-grained scale, resemble the Newtonian potential, yielding the first approximation to the metric structure of classical general relativity. In analogy with Consoli's model, taking into consideration the long-wavelength modes, here gravity is induced by the underlying field $\Phi(x)$ which describes the density fluctuations of the vacuum. In weak gravitational fields, on a coarse-grained scale, the underlying field $\Phi(x)$ can be identified with the Newtonian potential

$$\Phi \approx U_N = -G_N \sum_i \frac{M_i}{|\vec{r} - \vec{r}_i|}, \tag{12}$$

namely

$$g_{\mu\nu} = g_{\mu\nu}\left[\Phi(x)\right]. \tag{13}$$

An interesting argument which allows us to characterize the quantum microscopic structure of the underlying background generating gravity can be derived from the quantum uncertainty principle [15] and from the hypotheses of space-time discreteness at the Planck scale. In particular, in regard to the granularity of space-time and its link with gravity, in the papers [16-19] Ng showed that the quantum fluctuations of space-time manifest themselves in the form of uncertainties in the geometry of space-time and thus the structure of the space-time foam can be inferred from the accuracy with which we can measure its geometry. By considering a mapping of the geometry of space-time for a spherical volume of radius l over the amount of time $T = 2l/c$ it takes light to cross the volume, in Ng's approach the average separation between neighbouring cells of space corresponds to the average minimum uncertainty, and thus to the accuracy in the measurement of a distance l, given by

$$\delta l \geq \left(2\pi^2/3\right)^{1/3} l^{1/3} l_P^{2/3}. \tag{14}$$

An interesting aspect of Ng's quantum foam model lies in its holographic features in the sense that here, dropping the multiplicative factor of order 1, a spatial region of size l can contain no more than $l^3/\left(l l_P^2\right) = \left(l/l_P\right)^2$ cells and thus a maximum number of bits of information $\left(l/l_P\right)^2$ in agreement with the holographic principle [20-25] which implies that, although the world around us appears to have three spatial dimensions, its contents can actually be encoded on a two-dimensional surface, like a hologram.

By applying the discreteness hypothesis of Ng's model, namely the fact that we cannot make Δx smaller than the elementary length (14)[1]:

$$\Delta x \geq \cdot\left(2\pi^2/3\right)^{1/3} l^{1/3} l_P^{2/3} \tag{15}$$

to Heisenberg's uncertainty relation for the position Δx and momentum Δp

$$\Delta x \geq \frac{\hbar}{2\Delta p} \tag{16}$$

one obtains that, if Δp increases, the expression of Δx as a function of Δp must contain a term directly proportional to Δp that counterbalances the term proportional to $\left(\Delta p\right)^{-1}$. By following [26], a possible choice, at the first order in Δp, is:

$$\Delta x \geq \frac{\hbar}{2\Delta p} + \frac{\Delta p}{2\hbar}\left(2\pi^2/3\right)^{2/3} l^{2/3} l_P^{4/3} \tag{17}$$

in which the factor in the second term of the right hand side is selected by means of dimensional arguments. The expression (17) can be viewed as the "generalized" version of the uncertainty principle in a discrete space-time.

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[1] An analogous limitation holds in time.

By a similar reasoning one can obtain the corresponding version of (17) for time uncertainty as:

$$\Delta t \geq \frac{\hbar}{2\Delta E} + \frac{\Delta E T_0^2}{2\hbar}, \tag{18}$$

where ΔE is the energy uncertainty and $T_0 = \frac{1}{c}\left(2\pi^2/3\right)^{1/3} l^{1/3} l_P^{2/3}$ is the elementary time. In the approach proposed in this article, the new terms appearing in equations (17) and (18) have a very special meaning: they represent the "intrinsic" uncertainty of space-time due to the presence of a particle of a given energy–momentum deriving from opportune changes of the quantum vacuum energy density $\Delta\rho_{qvE} = \rho_{pE} - \rho_{qvE}$. Thus, the presence of matter of density (4) modifies the geometry of space-time. In fact, the energy $E \approx pc$ contained in a region of size L and deriving from matter of density (4) modifies the extension of this region of an amount:

$$\Delta L \cong \frac{\left(2\pi^2/3\right)^{1/3} l^{1/3} l_p^{2/3} T_0 E}{2\hbar}. \tag{19}$$

On the basis of equation (19), the curvature of space-time can be related to the presence of energy and momentum in it.

In other words, in the approach here suggested, one can say that the changes of the quantum vacuum energy density associated with the presence of matter of density (4) correspond to an underlying microscopic background geometry defined by equation (19).

Moreover, taking into account that in Ng's model the holographic space-time foam defined by equation (14) can be related to the cosmic scale if the average minimum uncertainty (14) corresponds to a maximum energy density

$$\rho = \frac{3}{8\pi}\left(l l_P\right)^{-2} \tag{20}$$

for a sphere of radius l that does not collapse into a black hole, namely

$$\rho = \frac{3}{8\pi}\left(R_H l_P\right)^{-2} \tag{21}$$

where R_H is the Hubble radius (which is the critical cosmic energy density as observed), hence derives that the terms in equations (17) and (18) representing the "intrinsic" uncertainty of space-time due to the changes of the quantum vacuum energy density can be themselves related to the cosmic scale. In particular, by taking account of equation (21), equations (17) and (18) at the cosmological scale respectively become

$$\Delta x \geq \frac{\hbar}{2\Delta p} + \frac{\Delta p}{2\hbar}\left(2\pi^2/3\right)^{2/3} R_H^{2/3} l_P^{4/3}, \tag{22}$$

$$\Delta t \geq \frac{\hbar}{2\Delta E} + \frac{\Delta E T_0^2}{2\hbar}, \tag{23}$$

where ΔE is the energy uncertainty and $T_0 = \frac{1}{c}\left(2\pi^2/3\right)^{1/3} R_H^{1/3} l_P^{2/3}$. Finally, equation (19) describing the link between the underlying microscopic structure of space-time and the curvature of space-time, at the cosmological level may be expressed as

$$\Delta L \cong \frac{\left(2\pi^2/3\right)^{1/3} R_H^{1/3} l_p^{2/3} T_0 E}{2\hbar}. \tag{24}$$

Now, after showing how the quantum microscopic structure of the underlying background generating gravity can be characterized and the important link of this microscopic structure with the cosmic scale, the next fundamental step is to make explicit the role of the quantum vacuum energy densities given by equations (1) and (3) (in particular, in order to derive the critical cosmic energy density (21) as observed).

3. THE CHANGES OF THE 3D QUANTUM VACUUM ENERGY DENSITY AS THE ORIGIN OF THE CURVATURE OF SPACE-TIME

The Planck energy density (2) is usually considered as the origin of the dark energy and thus of a cosmological constant, if the dark energy is supposed to be owed to an interplay between quantum mechanics and gravity. However, the observations are compatible with a dark energy

$$\rho_{DE} \cong 10^{-26} Kg/m^3 \tag{25}$$

and thus equations (2) and (25) give rise to the so-called “cosmological constant problem” because the dark energy (25) is 123 orders of magnitude lower than (2). In order to solve this problem, an interesting explanation for the actual value (25) which invokes the fluctuations of the quantum vacuum has recently been suggested by Santos [27-29]. According to this approach, quantum vacuum fluctuations determine a curvature of space-time and, under plausible hypotheses within quantized gravity, a relation between the two-point correlation function of the vacuum fluctuations and the space-time curvature was obtained. The quantum vacuum fluctuations can be associated with a curvature of space-time similar to the curvature produced by a "dark energy" density, on the basis of the equation

$$\rho_{DE} \cong 70G\int_0^\infty C(s)sds \tag{26}$$

which states that the possible value of the "dark energy" density is the product of Newton constant, G, times the integral of the two-point correlation function of the vacuum fluctuations defined by

$$C(|\vec{r}_1 - \vec{r}_2|) = \frac{1}{2}\langle vac|\hat{\rho}(\vec{r}_1,t)\hat{\rho}(\vec{r}_2,t) + \hat{\rho}(\vec{r}_2,t)\hat{\rho}(\vec{r}_1,t)|vac\rangle, \tag{27}$$

$\hat{\rho}$ being an energy density operator such that its vacuum expectation is zero while the vacuum expectation of the square of it is not zero. The correlation function (27) determines also the gravitational energy associated with the vacuum fluctuations according to the equation

$$\rho_{grav}c^2 = -4\pi G\int_0^\infty C(r_{12})r_{12}dr_{12}. \tag{28}$$

Moreover, dimensional analysis leads to Zeldovich's formula [11],

$$\rho_{DE}c^2 \approx \frac{Gm^2}{r_C}\cdot\frac{1}{r_C^3}, \tag{29}$$

($r_C = \hbar / mc$ being Compton's radius) which involves a parameter, m, with dimensions of a mass. If in Zeldovich's original model, equation (29) reproduces the observed value of the dark energy density for a mass of $m \approx 7{,}6\cdot 10^{-29}\,Kg$ that is about 1/20 times the proton mass or about 80 times the electron mass, Santos' approach does not allow to derive the value of m, but inside his approach it is plausible to assume that vacuum fluctuations of high energy, involving very massive particles, would not be probable.

Here, our aim is to show that the curvature of space-time associated with a dark energy density can be interpreted as a consequence of more fundamental changes of the 3D quantum vacuum energy density $\Delta\rho_{qvE} = \rho_{pE} - \rho_{qvE}$, in other words it can be physically defined as the mathematical value of the 3D quantum vacuum energy density (whose underlying microscopic structure is characterized by a geometry expressed by equations (17)-(19) and by equations (22-(24) at the cosmological level). In this regard, before all, we assume that the expectation value of the stress-energy tensor operator of the quantum fields (9) at any point gives the matter energy associated with the matter (baryonic plus dark) energy density, which is determined by changes and fluctuations of the 3D quantum vacuum energy density, without any additional contribution to the vacuum. This assumption allows us to obtain the correct Friedmann-Robertson-Walker metric

$$ds^2 = -dt^2 + \left[a(t)\right]^2 \left(dr^2 + r^2 d\Omega\right) \tag{30}$$

in which the recession of the distant galaxies can be calculated in terms of the link of the measurable Hubble constant and of the deceleration parameter with the time-dependent parameter $a(t)$), by introducing new time and radial coordinates r' and t' given by relations

$$r = \frac{r'}{a(t')} + O\left(r'^3\right), \; t = t' - \frac{r'^2}{2a(t')}\frac{da(t')}{dt'} + O\left(r'^4\right). \tag{31}$$

By inserting (31) into (30) one obtains the equation

$$\begin{aligned} ds^2 &= \left[1 + \left(\frac{\dot{a}}{a}\right)^2 r'^2\right] dr'^2 + r'^2 d\Omega^2 - \left[1 + \left(\frac{\dot{a}}{a}\right)^2 r'^2\right] dt'^2 = \\ &\left[1 + \frac{8\pi G}{3}\left(\rho_{mat} + \rho_{DE}\right) r'^2\right] dr'^2 + r'^2 d\Omega^2 - \left[1 - \frac{8\pi G}{3}\left(\frac{1}{2}\rho_{mat} - \rho_{DE}\right) r'^2\right] dt'^2, \end{aligned} \tag{32}$$

where the Friedmann equations

$$\left[\frac{\dot{a}}{a}\right]^2 = \frac{8\pi G}{3}\left(\rho_{mat} + \rho_{DE}\right), \; \frac{\ddot{a}}{a} = \frac{8\pi G}{3}\left(\frac{1}{2}\rho_{mat} t + \rho_{DE}\right) \tag{33}$$

have been taken into account in the second equality, $a \equiv a(t')$, $\dot{a} \equiv \frac{da(t')}{dt'}$, $\ddot{a} \equiv \frac{d^2 a(t')}{dt'^2}$, ρ_{mat} is the density of matter given by equation (4), ρ_{DE} is the density owed to a possible existence of dark matter. In reference to equation (32), the assumption that the expectation value of the stress-energy tensor operator of the quantum fields (9) gives the matter stress-energy density determined by fluctuations of the 3D quantum vacuum energy density, means that

$$\langle\Psi|\hat{T}_4^4|\Psi\rangle = \frac{\Delta\rho_{qvE}}{c^2};\ \langle\Psi|\hat{T}_\mu^\nu|\Psi\rangle \approx 0 \ \text{ for } \ \mu\nu \neq 00, \tag{34}$$

Ψ being the quantum state of the universe corresponding to the value of the field $\Phi(x)$ defining a given fine-grained history. This suggests to express the stress-energy tensor (9) corresponding to the quantum vacuum fluctuations as

$$\hat{T}_{\mu\nu}^{vac} \equiv \hat{T}_{\mu\nu} - \langle\Psi|\hat{T}_{\mu\nu}|\Psi\rangle \hat{I}, \tag{35}$$

where $\hat{I}$ is the identity operator. The existence of quantum vacuum fluctuations imply that, despite the expectation of $\hat{T}_{\mu\nu}^{vac}$ is zero by definition, one has

$$\langle\Psi|\hat{T}_{\mu\nu}^{vac}(x)\hat{T}_{\lambda\sigma}^{vac}(y)|\Psi\rangle \neq 0 \tag{36}$$

in general.

Now, in order to derive equation (32), taking into account Santos' results, inside our model it is reasonable to assume that the underlying quantum vacuum condensate can be characterized by considering the metric of the quantum vacuum defined by relation

$$d\hat{s}^2 = \hat{g}_{\mu\nu} dx^\mu dx^\nu, \tag{37}$$

where the coefficients (in polar coordinates) are

$$\hat{g}_{00} = -1 + \hat{h}_{00},\ \hat{g}_{11} = 1 + \hat{h}_{11},\ \hat{g}_{22} = r^2\left(1 + \hat{h}_{22}\right),\ \hat{g}_{33} = r^2 \sin^2\vartheta\left(1 + \hat{h}_{33}\right),\ \hat{g}_{\mu\nu} = \hat{h}_{\mu\nu} \ \text{ for } \ \mu \neq \nu \tag{38}$$

where multiplication of every term times the unit operator is implicit and, at the order $O(r^2)$, one has

$$\left\langle \hat{h}_{\mu\nu} \right\rangle = 0 \ \text{ except } \left\langle \hat{h}_{00} \right\rangle = \frac{8\pi G}{3}\left(\frac{\Delta\rho_{qvE}}{c^2} + \rho_{DE}\right) r^2 \ \text{ and}$$

$$\left\langle \hat{h}_{11} \right\rangle = \frac{8\pi G}{3}\left(\rho_{DE} - \frac{1}{2}\frac{\Delta\rho_{qvE}}{c^2}\right) r^2, \tag{39}$$

where $\left\langle \hat{h}_{\mu\nu} \right\rangle$ stands for $\langle\Psi|\hat{h}_{\mu\nu}|\Psi\rangle$ (and the fluctuations of the quantum vacuum $\Delta\rho_{qvE} = \rho_{pE} - \rho_{qvE}$ correspond to an underlying microscopic geometry defined by equations (17), (18) and (19)). In virtue of the quantized geometry defined by equations (17), (18) and (19), the metric (37), at a fundamental level, has to be considered as a quantized metric.

As regards the quantized metric (37), it is important to remark that in the approach developed by Santos in [28], by writing the quantum coefficients of the metric as (38), where

$$\left\langle \hat{h}_{\mu\nu} \right\rangle = 0 \text{ except } \left\langle \hat{h}_{00} \right\rangle = \frac{8\pi G}{3}\left(\rho_{mat} + \rho_{DE}\right) r^2$$

$$\text{and } \left\langle \hat{h}_{11} \right\rangle = \frac{8\pi G}{3}\left(\rho_{DE} - \frac{1}{2}\rho_{mat}\right) r^2 , \tag{39a}$$

where $\left\langle \hat{h}_{\mu\nu} \right\rangle$ stands for $\left\langle \Psi \middle| \hat{h}_{\mu\nu} \middle| \Psi \right\rangle$, in the approximation of the second order in the (small) tensor $\hat{h}_{\mu\nu}$, it is possible to derive the components of quantum Einstein equations of the form

$$\hat{G}_{\mu\nu} = \frac{8\pi G}{c^4} \hat{T}_{\mu\nu} . \tag{40}$$

In Santos' approach, the quantum Einstein tensor operator $\hat{G}_{\mu\nu}$ is expressed in terms of the operators $\hat{h}_{\mu\nu}$, by resolving these (non-linear coupled partial) differential operator equations (40) in order to obtain the quantum metric coefficients $\hat{g}_{\mu\nu}$ in terms of integrals involving the stress-energy tensor operator and finally calculating the expectations of the metric coefficients $\hat{g}_{\mu\nu}$ in terms of integrals involving the expectations of the stress-energy tensor operator. The reader can find details of these calculations, for example, in the above reference [28] and in [29].

Here, we underline that, in analogy with Santos' results, due to the fact that the relations between the metric coefficients and the matter stress-energy tensor are non-linear, the expectation of the quantized metric (37) of the vacuum condensate is not the same as the metric of the expectation of the matter tensor (9). The difference gives rise to a contribution of the vacuum fluctuations which reproduces the effect of a cosmological constant. Moreover, we will assume that, when the distance $r \to \infty$, one has $\hat{g}_{\mu\nu} \to \eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski metric.

By starting from the quantized metric (37) whose coefficients are defined by relations (38) and (39), one can obtain the components of the quantum Einstein equations (40) on the basis of the assumption that they are similar to the classical counterparts. In particular, the expectation value of the (operator) metric parameter $\hat{h}_{11}$ may be written in the form

$$\langle\Psi|\hat{h}_{11}|\Psi\rangle = \langle\Psi|\hat{h}_{11}|\Psi\rangle_{mat} + \langle\Psi|\hat{h}_{11}|\Psi\rangle_{vac}, \tag{41}$$

namely it is the sum of two expressions, one containing the matter density produced by the changes of the quantum vacuum energy density, and the other indicating the vacuum density fluctuations, ρ_{vac}. In equation (41), by modelling the matter density of the universe by means of a constant, the matter term can be expressed as

$$\langle\Psi|\hat{h}_{11}|\Psi\rangle_{mat} \cong \frac{2GM}{r} + \frac{2G^2M^2}{r^2}, \tag{42}$$

where $M = \frac{4}{3}\pi\rho_{mat}r^3 = \frac{4}{3}\pi\frac{\Delta\rho_{qvE}}{c^2}r^3$, which agrees with the second order expansion of the well-known Schwarzschild solution

$$g_{11} = \left(1 - \frac{2GM}{r}\right)^{-1}. \tag{43}$$

Moreover, taking into account equation (3), here the dark energy density ρ_{DE} can be associated with opportune fluctuations $\Delta\rho_{qvE}^{DE}$ of the 3D quantum vacuum energy density defined by relation

$$\Delta\rho^{DE}{}_{qvE} = \frac{m_{DE}\cdot c^2}{V}, \tag{44}$$

m_{DE} being the mass corresponding to the dark energy ρ_{DE} in the volume V and thus

$$\rho_{DE} = \frac{\Delta\rho_{qvE}^{DE}}{c^2}. \tag{45}$$

In this way, taking into account that according to Santos' results, the vacuum contribution appearing in equation (41), to order G^2, is

$$\langle\Psi|\hat{h}_{11}|\Psi\rangle_{vac} \cong 600G^2r^2\int_0^\infty C(s)sds, \tag{46}$$

r being a distance which is estimated to fulfil $r/s \approx 10^{40}$, in our model the vacuum contribution may be expressed as

$$\langle \Psi | \hat{h}_{11} | \Psi \rangle_{vac} \cong 150 \frac{1}{\pi} G^2 r^2 \left(\frac{V}{c^2} \Delta\rho_{qvE}^{DE} \right)^2 \frac{1}{l} \cdot \frac{1}{l^3}, \tag{47}$$

where

$$l = \frac{\hbar}{\left(\frac{V}{c^2} \Delta\rho_{qvE}^{DE} \right) c} \tag{48}$$

and, taking into account equation (26), Santos' integral of the two-point correlation function has been assimilated to the fluctuations of the quantum vacuum energy density (44) on the basis of equation

$$\left(\frac{V}{c^2} \Delta\rho_{qvE}^{DE} \right)^2 \frac{1}{l} \cdot \frac{1}{l^3} = 4\pi \int_0^\infty C(r_{12}) r_{12} dr_{12}. \tag{49}$$

Therefore, the total expectation value (41) becomes, to order r^2

$$\langle \Psi | \hat{h}_{11} | \Psi \rangle \cong \frac{8\pi G \Delta\rho_{qvE}}{3c^2} r^2 + 150 \frac{1}{\pi} G^2 r^2 \left(\frac{V}{c^2} \Delta\rho_{qvE}^{DE} \right)^2 \frac{1}{l} \cdot \frac{1}{l^3}. \tag{50}$$

Hence, a comparison with the Friedmann equations (33), taking account of relations (26) and (46), leads to the following equation

$$\rho_{DE} c^2 \cong \frac{35G}{2\pi V} \left(\frac{V}{c^2} \Delta\rho_{qvE}^{DE} \right)^2 \frac{1}{l} \cdot \frac{1}{l^3}, \tag{51}$$

namely

$$\rho_{DE} \cong \frac{35Gc^2}{2\pi \hbar^4 V} \left(\frac{V}{c^2} \Delta\rho_{qvE}^{DE} \right)^6 \tag{52}$$

which states the equivalence of the curvature of space-time produced by the changes of the quantum vacuum energy density and the one determined by a constant dark energy density. This means that in the approach based on equations (37)-(52), the changes and fluctuations of the quantum vacuum energy density generate a curvature of space-time similar to the curvature produced by a "dark energy" density. Moreover, it is interesting to observe that, while in

Santos' model, the dark energy is associated with the two-point correlation function of the vacuum fluctuations (on the basis of equation (26)), in the approach suggested by the authors of this article, the dark energy is directly determined by fluctuations of the quantum vacuum energy density on the basis of equation (52). It must be emphasized that here the fluctuations of the quantum vacuum energy density play the same role of Santos' two-point correlation function. In other words, there is an equivalence between the fluctuations of the quantum vacuum energy density and the two-point correlation function: in the approach here suggested, the fluctuations of the 3D quantum vacuum energy density act as a two-point correlation function on the basis of relation

$$\frac{c^4}{4\pi\hbar^4}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^6 \cong \int_0^\infty C(s)sds. \tag{53}$$

Moreover, introducing equation (52) into equation (39), the expectation values of the coefficients of the quantized metric (30) become

$$\left\langle \hat{h}_{\mu\nu}\right\rangle = 0 \text{ except } \left\langle \hat{h}_{00}\right\rangle = \frac{8\pi G}{3}\left(\frac{\Delta\rho_{qvE}}{c^2}+\frac{35Gc^2}{2\pi\hbar^4 V}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^6\right)r^2$$

$$\text{and } \left\langle \hat{h}_{11}\right\rangle = \frac{8\pi G}{3}\left(-\frac{\Delta\rho_{qvE}}{2c^2}+\frac{35Gc^2}{2\pi\hbar^4 V}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^6\right)r^2, \tag{54}$$

namely turn out to depend directly on the changes of the quantum vacuum energy density. As a consequence, one can say that the changes and fluctuations of the quantum vacuum energy density, through the quantized metric (37) of the quantum vacuum condensate whose coefficients are defined by equations (38) and (54) (and whose underlying microscopic geometry is described by equations (17)-(19) and, at the cosmological level, by equations (22)-(24)) can be considered the origin of the curvature of space-time characteristic of general relativity. In other words, one can say that the curvature of space-time may be considered as a mathematical value which emerges from the quantized metric (37) and thus from the changes and fluctuations of the quantum vacuum energy density (on the basis of equations (38) and (54)). In synthesis, according to the view suggested in this chapter, the quantized metric (37) associated with the changes and fluctuations of the quantum vacuum energy density, on the basis of equations (38) and (54), can be considered as the ultimate visiting card of general relativity.

4. ABOUT THE MOTION OF A MATERIAL OBJECT IN THE CURVED SPACE-TIME

Now, let us see how the curvature of space-time corresponding to the changes and fluctuations of the quantum vacuum energy density acts on a test particle of mass m_0, in other words how the motion of a material object in a background characterized by changes of its energy density can be treated mathematically. When a material object corresponding to a given diminishing of the quantum vacuum energy density moves, this diminishing of the quantum vacuum energy density – by virtue of its link with the quantum vacuum condensate defined by equations (54) (and whose underlying microscopic geometry is described by equations (17)-(19) and, at the cosmological level, by equations (22)-(24)) – causes a shadowing of the gravitational space which determines the motion of other material objects present in the region under consideration.

In the approach here suggested, the shadowing of the gravitational space determined by a variable density of quantum vacuum tries inspiration from the idea of the polarizability of the vacuum in the vicinity of a mass (or other mass-energy concentrations) introduced by Puthoff's polarizable model of gravitation [6]. In order to interpret and reproduce the curvature of space-time Puthoff postulated the following relation for the variable polarization of the vacuum caused by the presence of a mass

$$\vec{D} = K\varepsilon_0\vec{E}, \tag{55}$$

where $\vec{E}$ is the electric field, K is the (altered) dielectric constant of the vacuum (typically a function of position) due to (general relativistic-induced) vacuum polarizability changes under consideration. Puthoff's equation (55) establishes that the presence of electromagnetic energy or massive objects modulates the vacuum polarization in a linear fashion. The vacuum dielectric constant K constitutes the ultimate visiting card of Puthoff's model. Its effects on the various measurement processes that characterize general relativity are the following: the velocity of light changes from c to c/K, the time intervals change from Δt_0 to $\Delta t_0\sqrt{K}$ (which indicates that for $K>1$, namely in a gravitational potential, the time intervals between clock ticks is increased, that is the clock runs slower), the lengths of rods change from Δr_0 to $\Delta r_0/\sqrt{K}$. In Puthoff's model, the curvature of space – for example in the vicinity of a planet or a star – is associated with the effects on measurement processes of lengths and time intervals that take place for $K>1$. Such an influence on the measuring processes due to induced polarizability changes in the vacuum near the body under consideration leads to the general-relativistic concept that the presence of a body "influences the metric".

Trying inspiration from Puthoff's idea of polarizability of the vacuum in our model we assume that the shadowing (polarization) of the 3D quantum vacuum can be expressed by the equation

$$\vec{D} = \kappa\varepsilon_0\vec{E}_g, \tag{56}$$

where κ is a factor which represents the relatively small amount of the altered permittivity of the free space (with respect to the situation in which the energy density of quantum vacuum is given by equation (1)) and

$$\vec{E}_g = -H_{eg}\left(\frac{V}{c^2}\Delta\rho_{qvE} + \frac{35Gc^2}{2\pi\hbar^4}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^6\right)\frac{1}{r^2}\hat{r} \tag{57}$$

can be defined as the gravitostatic field determined by both density of matter and density of dark energy (here $H_{eg} = \frac{G}{c^2}$ is the basic gravitodynamic constant)[2]. The gravitostatic field is linked with the quantum vacuum condensate defined by equations (54) (and whose underlying microscopic geometry is described by equations (17)-(19) and, at the cosmological level, by equations (22)-(24)) through relation

$$\vec{E}_g = -\frac{3H_{eg}V}{8\pi G}\left\langle\hat{h}_{00}\right\rangle\frac{1}{r^2}\hat{r}. \tag{58}$$

The total lagrangian density for matter-field interactions in the polarized vacuum is given by relation

$$L_d = -\left(\frac{m_0c^2}{\sqrt{K}}\sqrt{1-\left(\frac{v}{c/K}\right)^2} + q\Phi - q\vec{A}\cdot\vec{v}\right)\delta^3\left(\vec{r}-r_0\right) - \frac{1}{2}\left(\frac{B_g^{\,2}}{K\mu_0} + K\varepsilon_0 E_g^{\,2}\right)$$
$$-\frac{\lambda}{K^2}\left(\left(\nabla K\right)^2 + \frac{1}{\left(c/K\right)^2}\left(\frac{\partial K}{\partial t}\right)^2\right), \tag{59}$$

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[2] In analogy with Sacharov's germinal proposal of treatment of gravitation as "metric elasticity" of space [3].

where $\left(\Phi, \vec{A}\right)$ are the gravitational potentials, $\vec{B}$ is the gravitomagnetic field defined by

$$\vec{B}_g = H_{eg} \frac{\vec{J}}{r^3}, \tag{60}$$

(where $\vec{J} = \vec{L} + \vec{S}$, $\vec{L} = r \times \left(\frac{V}{c^2} \Delta\rho_{qvE} + \frac{35Gc^2}{2\pi\hbar^4} \left(\frac{V}{c^2} \Delta\rho_{qvE}^{DE} \right)^6 \right) \vec{v}$, $\vec{S}$ being the spin angular momentum of the material object determined by the diminishing of the quantum vacuum energy density under consideration) and $\lambda = \frac{c^4}{32\pi G}$. It must be emphasized that also the gravitomagnetic field (60), by virtue of the link of the orbital angular momentum of the material object determined by the diminishing of the quantum vacuum energy density with the quantum vacuum condensate defined by equations (54) expressed by

$$\vec{L} = r \times \frac{3V}{8\pi G} \left\langle \hat{h}_{00} \right\rangle \vec{v}, \tag{61}$$

is itself associated with the quantized metric of the quantum vacuum condensate.

Now, in analogy with Puthoff's polarizable vacuum model of gravitation [6], variation of the action functional with respect to the test particle variables leads to the following equation of motion of a test material object of mass m_0 in the polarized 3D quantum vacuum:

$$\frac{d}{dt}\left[\frac{\left(m_0 \kappa^{3/2}\right)\vec{v}}{\sqrt{1-\left(\frac{v}{c/\kappa}\right)^2}} \right] = m_0\left(c^2 \vec{E}_g + \vec{v}\times\vec{B}_g\right) + \frac{m_0 c^2}{\kappa} \cdot \frac{1+\left(\frac{v}{c/\kappa}\right)^2}{2\sqrt{1-\left(\frac{v}{c/\kappa}\right)^2}} \cdot \frac{\nabla\kappa}{\kappa}. \tag{62}$$

Equation (62) shows that there are two forces acting onto the test particle of mass m_0: the Lorentz force due to the quantum vacuum energy density surrounding the object and a second term representing the dielectric force proportional to the gradient of the shadowing of quantum vacuum (56). The importance of this second term lies in the fact that thanks to it one can account for the gravitational potential, either in Newtonian or general relativistic form. It might be interesting to note that with $m_0 \to 0$ and $v \to \frac{c}{\kappa}$, as would be the case for a photon, the deflection of the trajectory is twice as the deflection of a slow moving massive particle. This is an important indication of conformity with general relativity.

Variation of the action functional with regard to the κ variable leads to the expression of the generation of the shadowing of the gravitational space within general relativity, owed to the presence of both matter and fields. The equation has three right-hand side terms:

$$\nabla^2 \sqrt{\kappa} - \frac{1}{(c/\kappa)^2} \cdot \frac{\partial^2 \sqrt{\kappa}}{\partial t^2} = \frac{-\kappa}{4\lambda}\left[P(\kappa) + Q(\kappa) + R(\kappa)\right]. \tag{63}$$

Here $P(\kappa)$ represents the change in space shadowing by the mass density associated with the object of mass m_0, with the vector $\vec{r}$ as the distance from the system mass centre:

$$P(K) = \frac{m_0 c^2}{\sqrt{K}} \cdot \frac{1 + \left(\frac{v}{c/K}\right)^2}{\sqrt{1 - \left(\frac{v}{c/K}\right)^2}} \cdot \delta^3\left(\vec{r} - \vec{r}_0\right). \tag{64}$$

$Q(\kappa)$ is the change caused by the energy density of the fields (57) and (60) determined by the diminishing of the quantum vacuum energy density:

$$Q(\kappa) = \frac{1}{2}\left(\frac{B_g^{\ 2}}{\kappa\mu_0} + \kappa\varepsilon_0 E_g^{\ 2}\right). \tag{65}$$

$R(\kappa)$ is the change caused by the quantum vacuum shadowing energy density itself:

$$R(\kappa) = -\frac{\lambda}{\kappa^2}\left((\nabla\kappa)^2 + \frac{1}{(c/\kappa)^2}\left(\frac{\partial\kappa}{\partial t}\right)^2\right). \tag{66}$$

In the case of a static gravity field of a spherical mass distribution (a planet or a star), the solution of equation (63) has a simple exponential form:

$$\sqrt{\kappa} = e^{GM/rc^2} \tag{67}$$

where $M = \dfrac{V\Delta\rho_{qvE}}{c^2}$. The solution (67) can be approximated by expanding it into a series:

$$\kappa = e^{2GM/rc^2} = 1 + \frac{2GM}{rc^2} + \frac{1}{2}\left(\frac{2GM}{rc^2}\right)^2 + \dots . \tag{68}$$

This solution reproduces (to the appropriate order) the usual general-relativistic Schwarzschild metric predictions in the weak field limit conditions (i.e. solar system).

According to this model, it is important to underline that also particles without mass (for example, photons) have an indirect influence on the quantum vacuum energy density. In fact, because of equation (65) also a photon will add a contribution to the effective curvature of space-time associated with the fields (57) and (60). This result turns out to be also in accordance with general theory of relativity, where both mass and energy cause the curvature of space-time.

Moreover, with the obtained solution (67) or (68) regarding the factor κ measuring the polarizability of the quantum vacuum in the presence of matter, one can analyze the gravitational red shift characteristic of general relativity, and find inside this approach a more detailed form in order to obtain the frequency shift of the photon emitted by an atom on the surface of a star of mass *M* and radius *R*. Just like in Puthoff's model, the photon detected far away from the star will appear red shifted by the following amount:

$$\frac{\Delta\omega}{\omega_0} = \frac{\omega - \omega_0}{\omega_0} \approx -\frac{GM}{Rc^2}, \tag{69}$$

where we have assumed $\dfrac{GM}{Rc^2} << 1$. The photon, after having climbed up the gravity potential of the star, will retain its acquired frequency unchanged, and the change in frequency can be tested locally by comparing it with photons emitted by the same type of atoms at the same temperature, but within the weak gravity field of the laboratory.

With that same result it is also possible to analyse the amount of the bending of light rays from a distant star passing near a massive body, like in the classic general relativity test performed by Eddington's expedition during the solar eclipse in May 1919. The light ray from a distant star, while passing close to the Sun, will experience a gradual slowing of wavefront velocity coming towards the Sun, and a gradual increasing velocity in leaving the Sun's gravity field. Because κ increases closer to a massive body ($\kappa > 1$), the velocity of light will vary as c/κ. The part of the wavefront closer to the Sun will thus experience a greater slowdown than the part of the wavefront passing further away. This is seen from the Earth as an apparent shift of the position of the star close to the Sun's disk edge in the outward direction. In general relativity's terms, this deflection is a measure of local space-time curvature. We are interested in calculating the total bending angle. Because in case of the Sun the total deflection is small ($\varphi < 2$ arc-seconds) we can apply the usual low angle approximations throughout the calculation. And because of the same reason we will not make a big mistake if we approximate the variable velocity of light to the first order term of the series expansion (66) of κ:

$$v = \frac{c}{\kappa} \approx \frac{c}{1 + \frac{2GM}{rc^2}} \approx c\left(1 - \frac{2GM}{rc^2}\right). \tag{70}$$

In this relation the radius-vector *r* denotes the distance of the wavefront from the centre of the Sun as it travels by from $-\infty$ to $+\infty$, with the minimum distance of $R+\delta$ where *R* is the Sun's radius, and δ is the minimum distance from the Sun's surface. By assigning *z* to the distance of the wavefront along the line of sight (perpendicular to $R+\delta$), the radius-vector becomes $r = \sqrt{\left(R+\delta\right)^2 + z^2}$, so the equation (70) can be written as:

$$v \approx c\left(1 - \frac{2GM}{c^2} \cdot \frac{1}{\sqrt{\left(R+\delta\right)^2 + z^2}}\right). \tag{71}$$

The differential velocity of light, assuming $\delta << R$, is then

$$\Delta v = \frac{2GM}{c^2} \cdot \frac{R\delta}{\left(R^2 + z^2\right)^{3/2}}. \tag{72}$$

As the wavefront travels a distance $dz \approx vdt$, the differential velocity along the path of light results in an accumulated wavefront path difference Δz:

$$\Delta z = \Delta v dt \approx \frac{2GM}{c^2} \cdot \frac{R\delta}{\left(R^2 + z^2\right)^{3/2}} dz. \tag{73}$$

This results in an accumulated tilt angle of:

$$\varphi \approx \Delta z / \delta \approx \frac{2GM}{c^2} \cdot \frac{R\delta}{\left(R^2 + z^2\right)^{3/2}} dz. \tag{74}$$

By integrating equation (74) over the entire path $-\infty < z < +\infty$ yields:

$$\varphi \approx \frac{4\pi GM}{Rc^2}. \tag{75}$$

By inserting $G = 6,672 \cdot 10^{-11} Nm^2 Kg^{-2}$, $M = 1,9891 \cdot 10^{30} Kg$, and $R = 6,96 \cdot 10^8 m$, we obtain $\varphi = 1,75$ arc-seconds, which is exactly the value predicted by Einstein's general theory of relativity in 1915, and experimentally verified by Eddington in 1919 (between 1.2 and 1.9 arc-seconds, mainly because of the imperfect optics of the portable telescopes used).

Moreover, as regards the equations of motion (62) and (63), it is important to emphasize that, according to this approach, the modification of the quantum vacuum energy density determining both the matter density and dark energy density and the action of the shadowed quantum vacuum on another material object are phenomena directly determined by the fields (60), (64), (65) and (66). This implies that no time is needed to transmit the information from a material object to the surrounding region in order to shadow the gravitational space because the change of the quantum vacuum energy density is already there as it is associated with the fields (60), (64), (65) and (66) (what propagates from point to point is just the actual effects of this change); and, on the other hand, that no time is needed to transmit the information from the shadowed gravitational space to another material object in order to cause its movement.

Finally, according to the view proposed here, the 3D quantum vacuum as a direct medium for the transmission of gravitation established by equations (64), (65) and (66) can express in an elegant mathematical way the perspective about the non-existence of gravitational waves. In this regard, it seems compatible with some Loinger's results according to which gravitational waves are only hypothetical and do not exist in the physical world [30, 31]. On the other hand, despite several attempts of research about the gravitational field performed since the 1960s (see for example the reference [32]), gravitational waves have not yet been detected. As underlined by Schorn in the paper [33], "To search for gravitational waves in a laboratory, classical or quantum mechanical detectors can be used. Despite the experiments of Weber (1960 and 1969) and many others (Braginskij et al., 1972; Drever et al., 1973; Levine and Garwin, 1973; Tyson, 1973; Maischberger et al., 1991; Abramovici et al., 1992; and Abramovici et al., 1996) and theoretical calculations and estimations (Braginski and Rudenko, 1970; Harry et al., 1996; and Schutz, 1997), gravitational waves have never been observed directly in laboratory".

It is also interesting to observe that recent NASA research confirms that universal space is flat with only a 0.4% margin of error which is a strong indication that curvature of space in general theory of relativity is only a mathematical description of energy density of universal space which originates in a more fundamental energy density of quantum vacuum [34]. NASA measurements regarding the geometry of universal space turn out to be completely in agreement with the approach developed in this paper.

5. CONCLUSIONS

A model of a three-dimensional quantum vacuum has been proposed in which the curvature of space-time emerges, in the hydrodynamic limit of some underlying theory of a microscopic structure of space-time, as a mathematical value of a more fundamental actual energy density of quantum vacuum. The fluctuations of the quantum vacuum energy density generate a curvature of space-time similar to the curvature produced by a "dark energy" density and produce a shadowing of the gravitational space which determines the motion of other material objects present in the region under consideration. In this approach, the interesting perspective is opened that the three-dimensional quantum vacuum acts as a direct medium of gravitation: at a fundamental level, no time (as duration) is needed to transmit gravity force. A given material object diminishes energy density of quantum vacuum which generates curvature of space-time. Gravity does not act directly between massive objects, gravity acts in the quantum vacuum: the changes of the quantum vacuum energy density cause curvature of space-time which generate gravitational attraction between massive bodies. This view does not require existence of hypothetical graviton as a "carrier" of gravity.

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